# Error vs Residual Analysis

Clamping helps observe that loss is quite blind to smaller errors in the range of 0 to 4, and so wouldn’t guide new points to those zones. They correspond to areas where gradients of u in t are highest. I want to follow this up with a similar map of curvature u\_xt at the same point and/or other geometric information used to guide resampling.

|  |
| --- |
| C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\Residual_vs_uError_NotClamped.pngC:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\Residual_vs_uError_ClampedAt4.png |
| Figure 1 – Comparison of PDE residual (the cost function value) vs actual error in U, averaged over 10 runs. |

# Gradient and Curvature Estimation through PINNs

15 000 Steps is the point at which the first re-sample is usually taken. Therefore it’s important that the model is capable of estimating the areas of high gradient / curvature correctly at 15 000 steps. This is shown in figure 2 and 3. All of this is based on a [2, 64, 64, 64, 1] tanh + glorot normal NN, using ADAM optimiser.

Figure 4 shows how well the model is capable of estimating the areas at the end of training (using the same PINNs but with adaptive resampling of points every 2000 steps based on residual information).

|  |  |  |  |
| --- | --- | --- | --- |
|  | Ground Truth | 2k Steps | 15k Steps |
|  | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\GroundTruth - u_x.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_2k_ux.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_15k_ux.png |
|  | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\GroundTruth - u_t.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_2k_ut.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_15k_ut.png |
| Figure 2 – Gradients obtained from PINNs vs Ground Truth | | | |

|  |  |  |  |
| --- | --- | --- | --- |
|  | Ground Truth | 2k Steps | 15k Steps |
|  | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\GroundTruth - u_xx.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_2k_uxx.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_15k_uxx.png |
|  | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\GroundTruth - u_tt.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_2k_utt.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_15k_utt.png |
|  | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\GroundTruth - u_tx.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_2k_uxt.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_15k_uxt.png |

Figure 3 – Curvature obtained through PINNs

|  |  |  |
| --- | --- | --- |
|  | Ground Truth | End Step |
|  | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\GroundTruth - u_x.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_end_ux.png |
|  | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\GroundTruth - u_t.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_end_ut.png |
|  | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\GroundTruth - u_xx.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_end_uxt.png |
|  | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\GroundTruth - u_tt.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_end_utt.png |
|  | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\GroundTruth - u_tx.png | C:\Users\mn17jilf\AppData\Local\Microsoft\Windows\INetCache\Content.Word\gradients_end_uxx.png |

# Error of PINNs using resampling via different methods

In almost every case initialising with Hammersley yields better accuracy. This does not increase time by any significant amount so it is done for all cases moving forward.

The main interest was looking at the error of different sources of information for guiding resampling. The baseline to compare against was Wu’s loss driven resampling, utilising hyperparameters of c=1 and k=1. First those hyperparameters were used, keeping the amount and frequency of resampling the same.

|  |  |  |
| --- | --- | --- |
| Method | Time (h) | Error |
| Residual, Random Initialisation | 6.62 | 4.923E-04 |
| Residual, Hammersley | 6.50 | 5.151E-04 |

The sources of information looked at were curvature (differentiating u in the x and t directions) was highest, as well as combinations of gradients in x and t (Sum, Average, Max and RMS). Curvature seemed best:

|  |  |  |
| --- | --- | --- |
| Method | Time (h) | Error |
|  | 6.67 | 1.11E-03 |
| Sum( | 6.46 | 1.97E-03 |
| Avg( | 6.78 | 1.61E-03 |
| Max( | 5.60 | 1.51E-03 |
| RMS( | 5.60 | 1.85E-03 |

The next step was to investigate was how a combination of using residual and geometric information would do. A very simple first investigation involved simply using loss for half of the resampling, then switching to using geometric information. As from the first results using and max( did best, these were used.

|  |  |  |
| --- | --- | --- |
| Method | Time (h) | Error |
|  | 6.56 | 6.34E-04 |
| Max( | 7.06 | 1.21E-03 |

However, as all these used the hyper-parameters that were found to work best for residual, I also wanted to look at doing a quick investigation varying these and seeing whether which method was best changed as a result. (c controls random distribution, k controls spread of biased points), These factors were changed one at a time and tested for the more successful methods to save time (, max( and residual->.

|  |  |  |  |
| --- | --- | --- | --- |
| Method | hyper parameter choice | Time (h) | Error |
| Residual | Default k=1,c=1 | 6.62 | 4.92E-04 |
| Residual | Default k=1,c=1 | 6.50 | 5.15E-04 |
|  | Default k=1,c=1 | 6.67 | 1.11E-03 |
| Max( | Default k=1,c=1 | 5.60 | 1.51E-03 |
|  | Default k=1,c=1 | 6.56 | 6.34E-04 |
| Residual | k=0.5, c=1 | 6.73 | 5.41E-04 |
|  | k=0.5, c=1 |  |  |
| Max( | k=0.5, c=1 | 7.08 | 6.84E-04 |
|  | k=0.5, c=1 | 6.72 | 3.83E-04 |
| Residual | k=1, c=5 | 6.04 | 1.50E-03 |
|  | k=1, c=5 |  |  |
| Max( | k=1, c=5 | 6.51 | 3.92E-02 |
|  | k=1, c=5 | 6.82 | 4.46E-04 |
| Residual | k=2, c=5 | 7.06 | 5.01E-02 |
|  | k=2, c=5 |  |  |
| Max( | k=2, c=5 | 6.65 | 3.05E-01 |
|  | k=2, c=5 | 6.78 | 5.44E-02 |

Yellow is better than residual on its own. It does involve residual. I need to check how it compares to the values Wu obtained, not just to my recreation of their method.

I also want to check the error history for these successful cases (would take too long to check for all of them). To see whether 100 rounds of resampling are really needed. Further on the main issue is that 100 resamples takes a very long time, and so checking accuracy using a much smaller number of resamples would be more relevant.

To Do:

Check error history. Check results of uxt hyperparameter investigation.